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System Dynamics in Inventory and Production Planning

An Introduction and Critical Overview

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Summary. The fundamentals of the modelling approach System Dynamics are described and illustrated by two simple inventory and production planning models. After discussing the structure of an implemented System Dynamics model the applicability of this modelling approach is critically analysed.

Zusammenfassung. Die Elemente der Modellierungskonzeption System Dynamics werden beschrieben und anhand von zwei einfachen Lager- und Produktionsplanungsmodellen demonstriert. Nach der Darstellung des Aufbaus eines implementierten System Dynamics Modells wird die Anwendbarkeit dieses Modellierungskonzeptes kritisch analysiert.

Introductory Remarks

System Dynamics is a well-known and often used approach of modelling and analysing dynamic systems. It allows the relatively easy development of models possessing some hundred or even several thousand variables. Such large models are acceptable since the formulation analysis and modification of System Dynamics models are based completely on the use of computer simulation. System Dynamics was originally developed by J.W. Forrester for the investigation of industrial systems [11]. It has since found application in many other areas. System Dynamics models are presently being used in all areas of economic and social sciences.[1]

This paper will restrict its investigation to the possibilities and limits of using System Dynamics for production and inventory planning, the field where it was first applied.

In the first section we present a short overview of the stages of a System Dynamics study. Then we will describe the formulation, analysis, and modification of System Dynamics models. In the next section the procedure for analysis and modification of a System Dynamics model will be reconstructed in the light of linear systems theory. This is followed by an examination of the Sprague Electric Company production and inventory model developed by Forrester. Finally, we shall attempt a critical appreciation of the value of System Dynamics for inventory and production system planning.

1. Stages in the Development and Application of System Dynamics Models

A System Dynamics study is comprised of the following stages: 1. formulation of a model; 2. validation of the model; 3. model analysis and model modification; and 4. implementation of the model. These stages will be briefly described to get a preliminary overview of the whole procedure.

(1) The formulation of a fully specified dynamic model is taken in three phases. First, a causal loop diagram of the system is developed. On the basis of this causal loop diagram, a flow chart is constructed, which expresses the level-rate interpretation of the System Dynamics approach. This diagram serves as a base for the formulation of a numerical specified dynamical difference equation model.

(2) The method for *validating* a System Dynamics model differs considerably from the procedures used, e.g. in econometrics. So Forrester rejects ex ante as well as ex post forecasting for model validation. In his view, a dynamic model is "validated" if it is an adequate representation of the "mental model" which the model user has formed about the system.

[1] For periodical accounts of System Dynamics developments see [19] and [5]. A comprehensive description of the System Dynamics conception of modelling is given in [10]

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(3) The third stage, *model analysis* and *modification*, differs, depending on whether a model has or has not exogenous variables. In the first case, the goal of this stage is "to find the forces of growth," in the second case, to analyse "the causes of fluctuation and instability" [10, p. 3–5]. Models with exogenous variables are analysed using a special method called test input response analysis. The model is set "artificially" in an equilibrium state and its behavior is simulated using a standardized time trajectory (test input) for the exogenous variables. The cause of fluctuation and instability and the possibility of eliminating system instability by changing decision rules are studied on the basis of the alternate test response of the model variables.

(4) The fourth stage concerns the *implementation* of a System Dynamics study. It can be characterized by the question: in which way is the result of the analysis and modification phase to be used to get a desired improvement in the described system? Forrester gives no clear answer. There is a range of possible degrees of implementation. At one end of the spectrum the model user gets a "better feeling" for the systems behavior, which will enable him (in some way) to improve the system. At the other end, decision rules, which have shown in the model analysis to bring about improvement, are directly implemented into the system.

In the following sections these stages are discussed in greater detail.

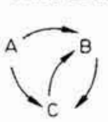
2. Formulation of System Dynamics Models

2.1. Formulation of Causal Loop Diagrams

The first phase in developing a System Dynamics model consists of the formulation of a *causal loop diagram*. The (vague) hypothesis "a change in variable A causes a change in variable B" can be graphically represented by

$$A \rightarrow B \qquad (1)$$

If the variables of such hypotheses are connected with each other, their graphical representation leads to a network of arrows as in the following example



If such a network contains a chain of arrows which form a closed cycle this cycle is called a causal loop or a feedback loop. A–B–C–A and B–C–B in the example above are such causal loops. Networks of arrows which contain at least one causal loop are causal loop diagrams. A causal loop diagram is a representation of a feedback system. One goal of a System Dynamics study is the representation of such feedback systems using numerically specified equations. A step in the direction of a more informative system representation is the development of a causal loop diagram with trend relations.

The hypothesis "an increase in A leads to an increase in B" contains more information than (1) and can be represented by

$$A \xrightarrow{+} B$$

In contrast to this "positive trend hypothesis," the "negative trend hypothesis" is: "an increase in A leads to a decrease in B." It is symbolized by

$$A \xrightarrow{-} B$$

A causal loop diagram is a visual model of a positive and negative trend hypothesis. Such causal diagrams allow us to distinguish between positive and negative feedbacks.

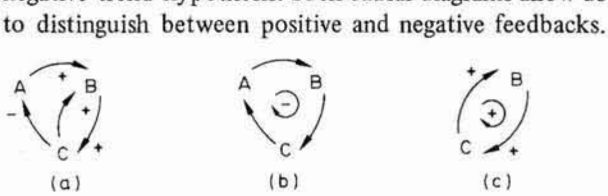


Fig. 1. Examples of positive and negative feedback loops

A *positive feedback loop* is a closed chain in which an increase in one variable causes a (delayed) increase in the same variable. In a *negative feedback loop* an increase in one variable induces a (delayed) decrease in the same variable. The causal diagram (a) in Fig. 1 contains one positive and one negative feedback loop. These loops are separately shown by the diagramms (b) and (c).

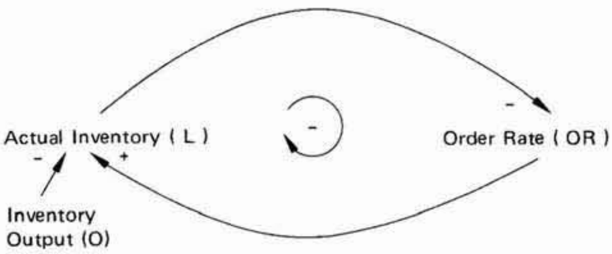


Fig. 2. Causal loop diagram of an inventory planning system

Figure 2 shows a causal loop diagram of a simple inventory planning system. The development of a causal loop diagram (normally) helps to determine the number of variables which should be included in the model. Forrester's objective is the development of *closed* models, i.e. models which do not have exogenous variables. He also admits models with exogenous variables, if these models are investigated by a special procedure (test input response analysis) which is discussed in Sect. 3. Although models with more than one exogenous variable are possible, nearly all System Dynamics models do not possess more than one exogenous variable. System Dynamics models with one exogenous variable can be called *singularly open* models. Figure 1 (a) shows a closed model, while the model in Fig. 2 is singularly open. According to Forrester's view, a model designer should develop a causal loop diagram of a system which (normally) does not contain more than one exogenous variable.

2.2. Formulation of Numerically Specified System Dynamics Models

Typically the next step would be the development of a System Dynamics diagram. This is a flow chart, which, like the causal loop diagram, is a visual model. In contrast to the causal loop diagram, its hypotheses are more precise and it shows the so-called level-rate interpretation of a system, which is an essential feature of the System Dynamics approach. As the development of System Dynamics diagrams presupposes a knowledge of different types of variables and hypotheses, let us first introduce them analytically. Afterwards, in Sect. 2.3, we will describe the formulation of System Dynamics diagrams.

2.2.1. Types of Model Relations

Using the System Dynamics approach the world is interpreted as a system of *levels* with inputs and outputs being connected in the form of feedback loops. If this network of feedback loops can be described in equations, the time-variant trajectories of all levels, inputs, and outputs can be deduced. This calculation is best done with a computer and is called simulation.

A level is described by a *level equation* which has the general form

$$L(t) = L(t-1) + DT[I(t-1,t)-O(t-1,t)] \quad \text{(units)}. \qquad (2)$$

Setting the time increment DT=1 it can be seen that the level value L at time t is a result of the level value of the previous period L(t−1), plus inflow I during the time period t−1 to t minus outflow O of the same time period.

The notation of System Dynamics models differs from this representation of a level equation. In this notation the level equation has the form

$$L.K=L.J+DT*(I.JK-O.JK) \quad \text{(units)}. \qquad (3)$$

The time subscripts of conventional difference equation notation (t+1, t and t−1) are replaced by L, K and J. Thus I.JK denotes the value of the inflow rate variable during J to K and O.JK the value of the corresponding outflow rate.

Accordingly, *rate variables* are always expressed by *rate equations* having the form

$$R.KL=F[L1.K,\ldots,LN.K] \quad \text{(units/time)}, \qquad (4)$$

where R is an inflow or outflow variable. From (3) and (4) we see that the time increment DT has the dimension (1/time). The time subscript KL corresponds to (t,t+1); F[...] expresses any given functional relationship, and L1,...,LN are the level variables of the model.

A complete System Dynamics model can be constructed using only level and rate equations (3) and (4). However, in order to make model relationships more visible, *auxiliary variables* are often included in the model. These are expressed by *auxiliary equations* having the general form

$$A.K=F[A1.K,A2.K,\ldots,AM.K,L1.K,L2.K,\ldots,LN.K]. \qquad (5)$$

Auxiliary variables can have the dimension of a level or a rate. The use of auxiliary equations enlarges the general form of rate equations to

$$R.KL=F[L1.K,\ldots,LN.K,A1.K,\ldots,AM.K] \quad \text{(units/time)}. \qquad (6)$$

In order to calculate the time path of the variables in a System Dynamics model, the initial values must be defined for each level. *Initial values* of a level are specified by *initial value equations*

$$L = \text{(numerical value)} \quad \text{(units)}. \qquad (7)$$

Likewise, *constants* must be specified by *constant equations*

$$C = \text{(numerical value)}. \qquad (8)$$

We shall now illustrate the different form of equations by modelling a very simple System Dynamics model of an inventory system.

The state of inventory is expressed by the level equation

$$L.K=L.J+DT*(I.JK-O.JK) \quad \text{(units)}, \qquad (9)$$

where I is the inflow and O the outflow rate of the inventory.

We will assume an initial stock of 500 units

$$L=500 \quad \text{(units)} \qquad (10)$$

and a constant outflow rate

$$O.KL=100 \quad \text{(units/week)}. \qquad (11)$$

The inflow rate is determined by the ordering rule (rate equation)

$$I.KL=100+AF*(DL-L.K) \quad \text{(units/week)}, \qquad (12)$$

Table 1. Elements of System Dynamics models and their DYNAMO expression

Model Element	Identification Letter	Equation Type
Level	L	L.K=L.J+DT*(I.JK−O.JK)
Rate	R	R.KL=F[L1.K,...,LN.K,A1.K,...,AM.K]
Auxiliary equation	A	A.K=F[L1.K,...,LN.K,A1.K,...,AM.K]
Initial level value	N	N = (numerical value)
Constant	C	C = (numerical value)

where DL denote the desired inventory level which will be specified by the constant equation

$$DL=300 \quad \text{(units)}. \tag{13}$$

AF is the factor which determines the amount of influence that the deviation between DL and L will have upon the amount of ordered material (I). It is specified as

$$AF=0.1 \quad \text{(1/week)}. \tag{14}$$

The difference between the desired inventory (DL) and the actual inventory (L) is defined by the auxiliary equation

$$D.K=DL-L.K \quad \text{(units)}. \tag{15}$$

Hence, from (12) and (15) we get the final form of I

$$I.KL=100+AF*D.K \quad \text{(units/week)}. \tag{16}$$

Equations (15) and (16) are examples of the general Eq. (5) and (6). Summarizing, all equations of the inventory model are given in System Dynamics notation (left) and conventional difference equation notation (right)

$$\begin{array}{ll} L.K=L.J+DT*[I.JK-O.JK] & L(t)=L(t-1)+DT*[I(t)-O(t)] \\ L=500 & L(0)=500 \\ O.KL=100 & O(t+1)=100 \\ I.KL=100+AF*D.K & I(t+1)=100+AF*D(t) \\ D.K=DL-L.K & D(t)=DL-L(t) \\ DL=300 & DL=300 \\ AF=0.1 & AF=0.1. \end{array} \tag{17}$$

The System Dynamics notation largely coincides with the notation of the computer simulation language DYNAMO. This language was specifically developed for programming System Dynamics models. In DYNAMO each equation is specified by its type, using a letter which preceeds the statement. Level equations are identified with an "L," auxiliary equations with an "A," rate equations with an "R," initial equations with an "N," and constant equations with a "C." Adding these instructions and four direction statements to (17), we get a complete DYNAMO program

```
L L.K=L.J+DT*(I.JK-O.JK)
N I=500
R O.KL=100
R I.KL=100+AF*D.K
A D.K=DL-L.K
C DL=300
C AF=0.1
SPEC DT=1,PRTPER=1,PLTPER=1,LENGTH=20
PRINT L,O,I
PLOT L/O/I
RUN
```
(18)

The last four statements include the specification and output instructions which determine the length of the simulation run and the form of the output (see [22]). The equation types are summarized in Table 1.

2.2.2. Special Kinds of Hypotheses in System Dynamics Models

System Dynamics models make considerable use of three types of hypotheses: the exponential delay hypothesis, the smoothing hypothesis, and the table function hypothesis. These are provided in DYNAMO as built-in macro instructions whose use is described below.

2.2.2.1. Exponential Delay Hypotheses. The exponential delay hypothesis used by Forrester describes the delayed relation between a level input (I) and a level output (O). Let us assume that a level L possesses an initial value L=0. The inflow of this level is assumed to be a unit pulse, i.e.

$$I(t) = \begin{cases} 1 & \text{for } t=0 \\ 0 & \text{for } t=1,2,\ldots \end{cases}$$

The time trajectory of O(t) is called a unit pulse response. Exponential delay levels are completely characterized by their pulse response function

$$O_p(t)=g(t) \quad t=0,1,2,\ldots$$

and

$$\sum_{t=0}^{\infty} g(t) = 1 \tag{19}$$

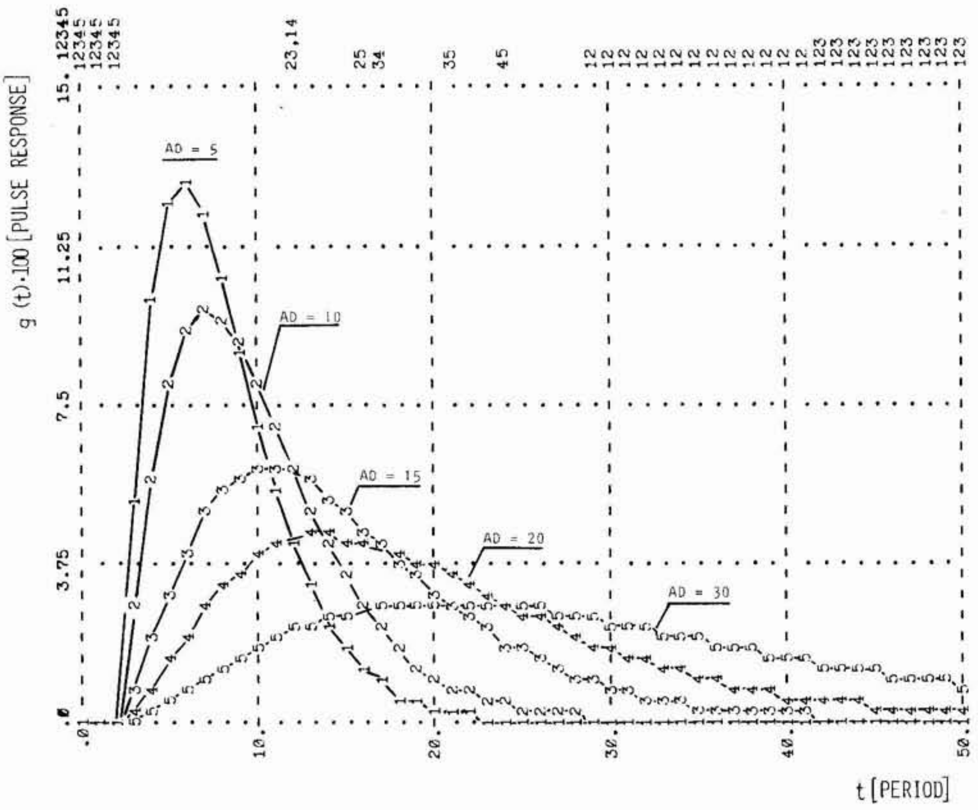


Fig. 3. Pulse response of an exponential third order delay with different average delay time (AD), I(0)=100

Forrester uses predominantly *third order exponential delays.* These delays have a unimodal pulse response and are simply specified by their *average delay* (AD). AD specifies the average time spent by an input element in the level (i.e. before leaving the level as an output). A third order exponential delay is expressed in DYNAMO by the *DELAY3 function*[2]:

R O.KL=DELAY3(I.JK,AD) (20)

Figure 3 shows the test response function of a third order exponential delay with different values of AD. If the model builder concludes that the relation between the inflow and outflow of a level can be described by a third order exponential delay, then the complete modelling of that delay is achieved by specifying its average delay (AD). (The identification of an observed inflow-outflow relation as a third order exponential delay is a critical point which will be discussed in Sect. 7.) Exponential third order delays contribute significantly to the dynamic character of System Dynamics models.

2.2.2.2. Smooth Hypotheses. In dynamic models variables are often used to express forecasts. In System Dynamics models Forrester uses such "forecasting variables" which are nearly exclusively in form of exponential smoothing forecasts. To explain such forecasting variables DYNAMO provides a built-in *SMOOTH function.*

Assuming the actual outflow of an inventory is called O, then it is reasonable to compute with the values of O an estimation (EO) for the next period lying in the future. According to Forrester, this estimate is to be made by an exponential smoothing forecast procedure which can be modelled in DYNAMO by the instruction

A EO.K=SMOOTH(O.JK,S) (21)

This SMOOTH function can be substituted by the normal DYNAMO equation

L EO.K=EO.J+DT*(O.J–EO.J)/S (22)
N EO=0

2 The DELAY3 function seems to contradict the rate equation (6) since the System Dynamics concept does not permit the direct influence of one rate variable by another. This contradiction is only apparent, however, because the DELAY3 macro instruction is shorthand for a chain of cascading levels [22, p. 92], in which I.JK of (20) is the inflow of a level and O.KL is explained by an auxiliary variable. The same holds for the smooth function (21) below

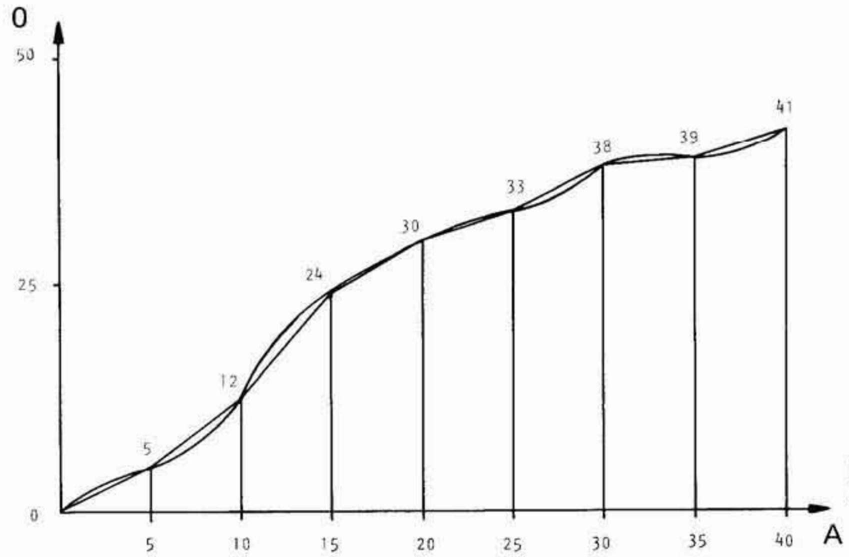


Fig. 4. Approximation of a non-linear relationship by an open polygon

We recognize that the *smoothing time* (S) is the reciprocal of the so-called smoothing constant, which is normally used in the discussion of forecasting methods.

2.2.2.3. Table Function Hypotheses. The non-linearity of System Dynamics models is often caused by *table functions.* These are open polygons with which arbitrary functional relationships can be approximated, as in the graph above (Fig. 4).

The open polygon, or table function, of Fig. 4 is expressed in a DYNAMO instruction as

A O.K=TABLE(TAB,A.K,0,40,5)
T TAB=0/5/12/24/30/33/38/39/41 (23)

(For details see [22, p. 31].) DELAY3 and SMOOTH functions will be demonstrated in Sect. 4. The use of table functions is described in Sect. 6.1.

2.2.3. Is-Ought Decision Rules

Knowing the structural form of the rate and auxiliary equation, we can discuss a type of decision rule which is a special characteristic of the System Dynamics approach. This type of decision rule (which is not explicitely named by Forrester) could be called the *is-ought decision rule.*

Virtually all of the decision rules used by Forrester to control inventory, production, and work force are is-ought decision rules. The concept underlying this kind of decision rule is to control the system by keeping the difference between desired levels (ought) and actual levels (is) as small as possible, in Forrester's words,

"A policy or rate equation recognizes a local goal toward which that decision point strives, compares the goal with the apparent system condition to detect a discrepancy and uses the discrepancy to guide action." Commenting on the rate equation OR=1/AT(DI–I): "In this equation the goal is the desired inventory DI. The order rate (OR) acts to move inventory towards the goal. The observed condition of the system is the inventory I... The action in the above rate equation is stated to be 1/AT of the discrepancy" (DI–I). [10, p. 415–416]

In the is-ought decision rules used by Forrester to specify a (decision) rate variable (RAT), the difference between the target DL and the actual value of a level L is explicitly stated, i.e. an is-ought decision rule always includes the term DL–L. A deviation of the level variable L from DL induces a change of RAT which tends to a reduction of this deviation. In many cases a linear is-ought decision rule is used having the form

$$\text{RAT.KL} = \text{EXSR.K} + \frac{1}{\text{APT}}\,(\text{DL1.K} - \text{L1.K} + \text{DL2.K} - \text{L2.K} + \ldots + \text{DLN.K} - \text{LN.K}) \quad (24)$$

EXSR is an exponentially smoothed rate variable (usually the firm sales rate). Variables DL1,...,DLN express target values of levels. 1/APT is a "compensation term," which balances out deviations between the target values and their corresponding actual values L1,...,LN. The decision variable (RAT) is the inflow rate of a level cascade of L1,...,LN. The desired values are generally defined by

$$\text{DLI.K} = \text{DF} * \text{EXSR.K} \quad (\text{I}=1,2,\ldots,\text{N}) \quad (25)$$

in which DF can be called *depletion factor* when the levels are inventories or backlogs.

Combining (24) and (25), the is-ought decision rule has the general form

$$RAT.KL = EXSR.K + \frac{1}{APT} (DF1*EXSR.K-L1.K+ \\ \ldots +DFN*EXSR.K-LN.K) \quad (26)$$

Since Forrester proposes a mandatory use of is-ought decision rules, they are central to his concept of system control. The problem concerning these rules is discussed later.

2.3. Development of System Dynamics Diagrams

The causal loop diagram (introduced in Sect. 2.1) serves as the base for developing of a *System Dynamics diagram* (often simply called flow chart). Figure 5 shows the diagram symbols for the variables and parameters.

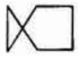
Model Element	Symbol
Level variable	
rate variable	
auxiliary variable	
initial level value	no symbol
constant	

Fig. 5. Diagram symbols of System Dynamics variables and parameters

Figure 6 shows the System Dynamics diagram of the simple inventory model (18). The actual inventory (L) has an input flow which is represented by the valve symbol I. It shows in a visual way that the amount of the inflow stream (I) depends upon the variable (D) and the parameters (AF and 100), from which a dotted line leads to the valve symbol I. The auxiliary (D) depends on the values of L and DL, and the outflow (O) of the inventory (L) has a constant amount of 100 units. We can recognize that a System Dynamics diagram is less precise than a DYNAMO model. For example, we recognize in the diagram which parameters and variables influence I, but the diagram does not express the form of the numerical specified Eq. (12).

The following generalizations about diagramming can be drawn from Fig. 6:

1) A dotted line shows that a level, an auxiliary, or a constant influences an auxiliary, a level, or a rate. The direction of the arrow is from the influencing to the influenced element.

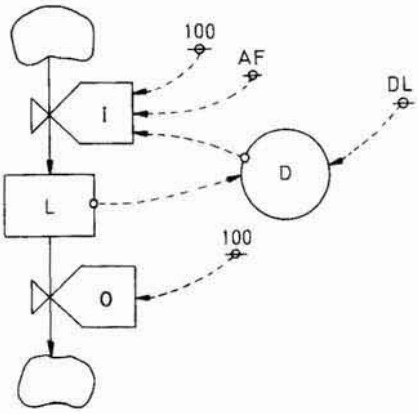
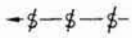


Fig. 6. System Dynamics diagram of a simple inventory model

2) Level inflows and outflows are shown by solid lines passing through valve symbols. Flows leading to levels, which are not included in the model, end in sinks, symbolized by cloud-like shapes. Flows from levels not included in the model are shown as originating in sources which are symbolized in the same way:

Model Element	Diagram Symbol	Use of Symbol
Sink		End of a flow (from a level)
Source		Beginning of a flow (to a level)
Dotted arrow		Arrow-head, showing direction of influence

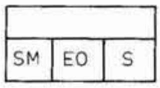
Specially marked flow lines can be used to show the kind of flow (i.e. inputs and outputs between the levels). Forrester, for example, uses the following flow symbols in his model of firms

	work force
	orders
	money
	material

The two built-in hypotheses described previously have the following diagram symbols, which are special versions of the common level symbols.

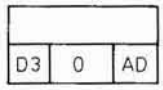
Smooth function

EO.K=SMOOTH(O.JK,S)



Delay3 function

O.KL=DELAY3(I.JK,AD)



3. Model Analysis and Model Modification

Forrester's objective is the study of closed systems. The essential idea of a closed system "is the boundary across which nothing flows (except perhaps a disturbance for exiting the system so we can observe its reaction)" [10, p. 4–2]. Closed systems are described by models which have no exogenous variables, i.e. by closed models. It seems as if Forrester would restrict his approach to systems which can be described by closed models. Looking at what Forrester means by a flow in the form of a disturbance, we will find, studying his models, that he admits (besides closed models) models with one exogenous rate variable. These singularly open models are not used for forecasting by specifying an estimation of the exogenous rate variable time trajectory. They are only investigated by a method which can be called *test input response analysis.* This method is borrowed from classical servomechanism theory. It consists of the following procedure:

(1) A singularly open System Dynamics model is "artificially" brought to a state of equilibrium by choice of initial level values.
(2) The time trajectory of the exogenous variable in the model is chosen to be a standardized test input.
(3) The test response of some relevant variable of the model is simulated.
(4) By more or less systematic variation of controllable model parameters one attempts to achieve a desired test response of the relevant variables.
(5) If the variation of special parameters does not lead to the desired behavior, different decision rules may be used to achieve this behavior, or one may even redesign whole sections of the model.

For test functions it is possible to use pulse, step, or sine functions, or certain random sequences of stochastic variables. In the following, step functions will be used only. Their definition is

$$S(t) = \begin{cases} SH > 0 & \text{for } t=N, N+1, \ldots \\ 0 & \text{for } t=N-1, N-2, \ldots \end{cases}$$

and the corresponding DYNAMO instruction is

STEP(SH,N)

After the discussion of the test input response analysis, we see that, in principle, it is possible to develop models which contain more than one exogenous (test input) variable. But a test input response analysis with more than one test input leads to considerable difficulties in the performance and analysis of the results (see [11, p. 141]). So Forrester recommends that "as a practical matter we are usually limited to one exogenous nonnoise test input!" [11, p. 141].

Since all System Dynamics models (known to the author) have none or only one exogenous variable, the use of System Dynamics models seems to be factually restricted to closed and singularly open models.

It is very difficult to describe an inventory and production planning system as a closed model because the ordering rate of the customer is normally interpreted as an exogenous variable. So all known System Dynamics models of inventory and production control are singularly open.

Using the test input response analysis for the improvement of singularly open inventory and production models, we can ask for the objective of such an improvement. In his study of the Sprague Electric Company's inventory and production system (discussed in Sect. 6 below) Forrester remarks, "the objective is to attain greater labor stability, less tendency for the system to amplify certain critical frequencies of external disturbances and less tendency for the system to be perturbed by internal or external random variations" [11, p. 176]. Applying these remarks to production and inventory systems in general, the goal of System Dynamics studies is to introduce production and inventory systems decision rules which flatten out the internally generated cycles of variables such as work force, production rates, etc. [13, p. 51].

4. A System Dynamics Study of Two Inventory and Production Planning Systems

Let us demonstrate now the two steps of model formulation and model modification by using two inventory and production planning models.

4.1. Inventory and Production Planning Model I

4.1.1. Formulation of the Model

Figure 7 shows the System Dynamics diagram of an inventory and production planning model (model I), being defined by the following set of equations.

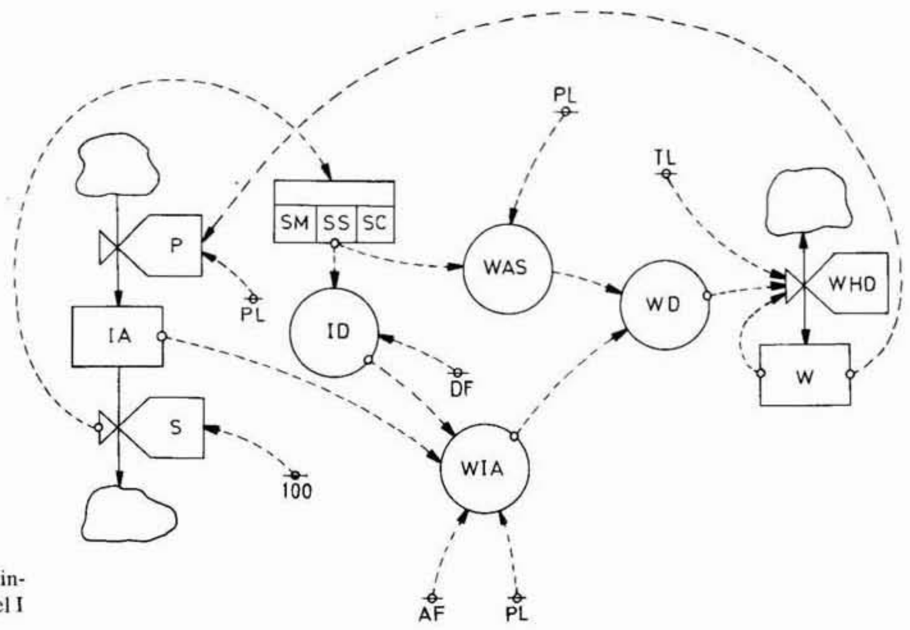


Fig. 7. System Dynamics diagram of inventory and production planning model I

The inventory is defined as

L IA.K=IA.J+DT*(P.JK–S.JK)

IA – Actual inventory (units)
P – Production rate (units/month)
S – Sales rate (units/month)

Since there is no under- or overemployment, the production rate is

A P.KL=W.K*PL

W – Work force (men)
PL – Productivity of work force (units/man-month)

The work force is defined as

L W.K=W.J+DT*WHD.JK

WHD – Work force hiring and dismissal rate (men/month).

The work force change rate is determined by an is-ought decision rule:

R WHD.KL=(WD.K–W.K)/TL

WD – Work force desired for production (men)
TL – Time for work force change (months).

Two variables determine the desired work force

A WD.K=WAS.K+WIA.K

WAS – Work force needed for average sale (men)
WIA – Work force needed for inventory adjustment (men)

where

A WAS.K=SS.K/PL

SS – Smoothed sales (units/month)

and

A WIA.K=(ID.K–IA.K)/(AF*PL)

ID – Inventory desired (units)
AF – Inventory adjustment factor (months).

The exponential smoothing of S results in

A SS.K=SMOOTH(S.JK,SC)

SC – Smoothing time constant (months).

Desired inventory (ID) is

A ID.K=DF*SS.K

DF – Inventory depletion factor (months).

If we assume that S has a constant inflow of 100 units and, given the parameters PL=5.2, SC=2, TL=4, AF2, and DF=1, then

R S.KL=100
C PL=5.2/SC=2/TL=4/DF=1/AF=2.

The initial values of W and L are specified as

N W=40
N IA=150.

The model is now complete.

4.1.2. Model Analysis and Modification

Equilibrium of the model can be achieved by the appropriate choice of initial level values. Let us demonstrate this for our model.

Variables being in equilibrium are identified with a dash above them. When our model is in equilibrium, the following conditions are satisfied:

$$\overline{S} = \overline{P} \quad (27)$$

$$\overline{P} = \overline{W}*PL \quad (28)$$

$$\overline{WHD} = 0$$

$$\overline{WD} = \overline{W}$$

$$\overline{WD} = \overline{WAS} + \overline{WIA}$$

$$\overline{WAS} = \overline{SS}/PL$$

$$\overline{SS} = \overline{S} \quad (29)$$

$$\overline{WIA} = 0$$

$$\overline{ID} = \overline{IA} \quad (30)$$

$$\overline{ID} = DF*\overline{SS} \quad (31)$$

First, the initial equilibrium level values of IA and W, which are dependent on the exogenous variable S, must be determined. Equations (29) to (31) imply

$$\overline{IA} = DF*\overline{S}.$$

From (27) and (28) it follows that the equilibrium value of W

$$\overline{W} = \overline{S}/PL.$$

The above two equations are expressed in DYNAMO as:

N IA=DF*S (32)

N W=S/PL. (33)

Before starting a test input response analysis, we now have to replace the initial values of IA and W with Eqs. (32) and (33). In addition, the rate equation for S in our model is exchanged by the step input specification

R S.KL = 500 + STEP(50,5)

These changes lead to the following "step input response-version" of the model

```
L IA.K=IA.J+DT*(P.JK-S.JK)
N IA=DF*S
R P.KL=W.K*PL
L W.K=W.J+DT*(WHD.JK)
N W=S/PL
R WHD.KL=(WD.K-W.K)/TL
A WD.K=WAS.K+WIA.K
A WAS.K=SS.K/PL
A WIA.K=(ID.K-IA.K)/(AF*PL)
A SS.K=SMOOTH(S.JK,SC)
A ID.K=DF*SS.K
R S.KL=500+STEP(50,5)
C PL=5.2/SC=2/TL=4/AF=2/DF=1
SPEC DT=1,LENGTH=50,PLTPER=1
PLOT IA=I,P=P
RUN
```

Case 6 in Fig. 8 shows the step response of the inventory IA (plot symbol "I") and the production rate P (plot symbol "P"). We can see that the system has internally generated oscillations. As mentioned before, the objective is to find decision rules which flatten out these oscillations. Let us assume that AF, SC, and TL are the controllable parameters of the system. Consequently, we have to change these parameters such that the step response of P and IA shows a stronger damping.

Figure 8 shows the results of five other simulations of the step response of IA and P using various parameter values. Of these, Case 1 shows the least internal oscillation and is therefore preferable. If model I were describing a real system, the decision rules for determining production and work force of case 1 could now be introduced into the real system.

4.2. Inventory and Production Planning Model II

4.2.1. Development of the Model

Changing parameters is only one way of improving the behavior of a System Dynamics model. In addition, decision rules may be replaced, or entire sections of the model redesigned. When it is found that the test input response of an inventory model oscillates strongly, regardless of the controlled parameter values used in the decision rules, one must consider structural changes in the described systems which would desirably alter the system's behavior. If a change in the organization structure promises to be successful, this change is introduced in the model, and the test response of the modified model simulated. If the test response shows better damping of oscillation, then the organizational changes in the model may be implemented in the real system. This procedure is of central importance for System Dynamics and is illustrated as follows in the example of a non-linear inventory and production planning model (model II). The System Dynamics diagram of model II is shown in Fig. 9. The inventory level is given by

L IA.K=IA.J+DT*(IO.JK−SA.JK)

IA – Inventory actual (units)
IO – Incoming orders (units/week)
SA – Actual sales (units/week).

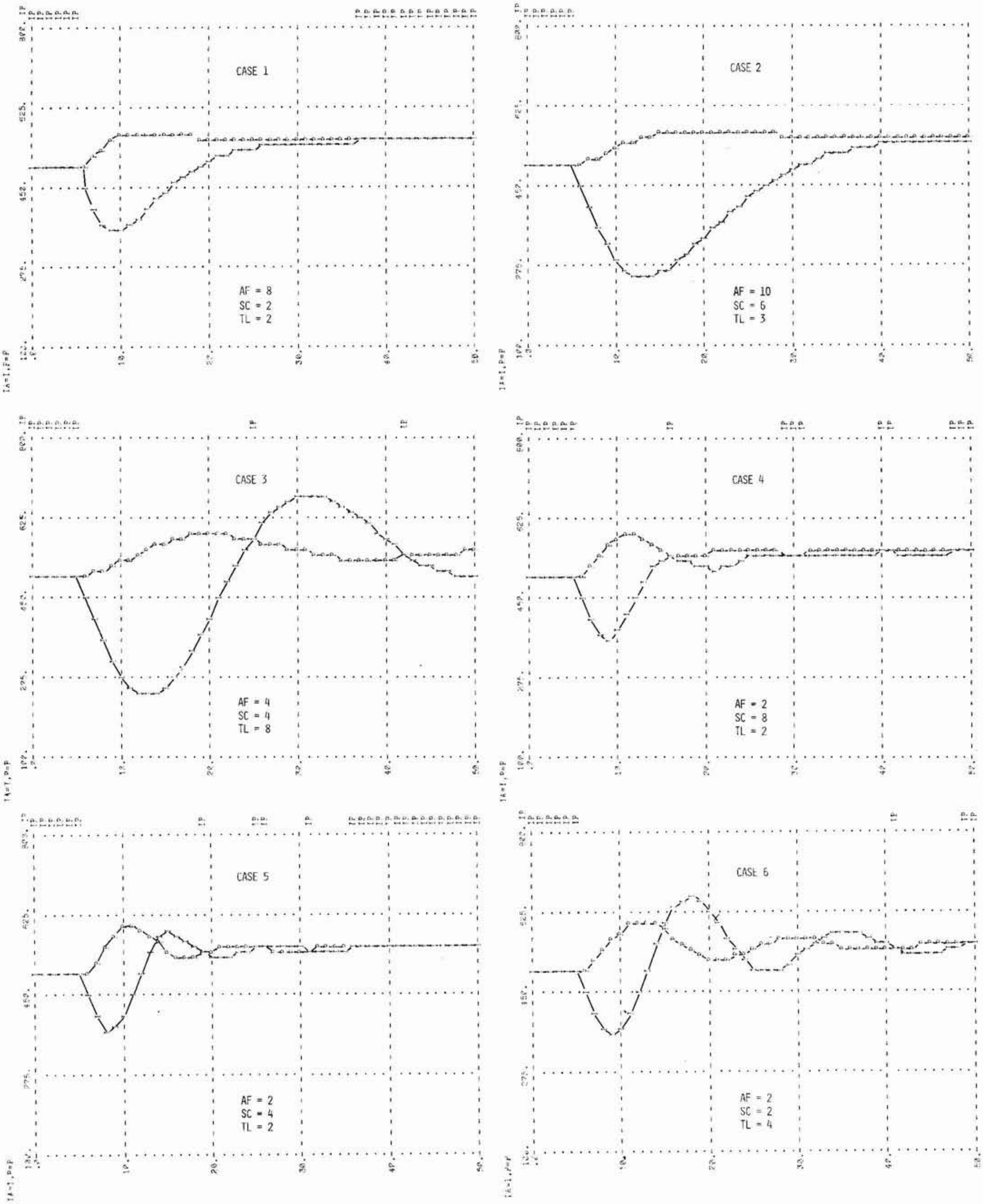


Fig. 8. Step response of inventory and production planning model I with different degrees of internal oscillation of production rate (P) and inventory level (I)

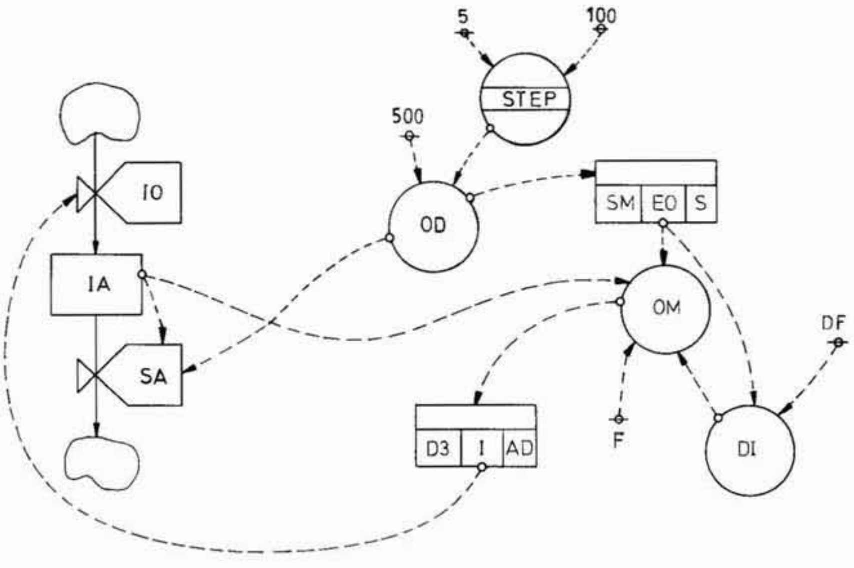


Fig. 9. System Dynamics diagram of inventory and production planning model II

The actual sales are limited by the actual inventory (IA) and the desired inventory outflow (OD)

R SA.KL=MIN(IA.K,OD.K)

OD – Desired inventory outflow (units).

The macro instruction MIN(IA.K,OD.K) compares IA and OD in each period K and chooses the lower value of both to specify SA.KL.

This relation causes the nonlinearity of the model. Ordered material (OM) is determined by the is-ought decision rule

R OM.KL=EO.K+F*(DI.K–IA.K)

OM – Ordered material (units/week)
EO – Estimated inventory outflow (units/week)
DI – Desired inventory (units)
F – Inventory adjustment factor (1/week).

According to (25) the desired inventory is

A DI.K=DF*EO.K

DF – Inventory depletion factor (week)

and

A EO.K=SMOOTH(OD.K,S)

S – Smoothing constant (weeks).

An exponential third order delay with an average delay (AD) of 8 weeks, is assumed to exist between the ordering of material (OM) and the inflow of the delivered material

R IO.KL=DELAY3(OM.JK,AD)

Before a test input response analysis can be made, the system must be brought into equilibrium. The initial equilibrium level value ($\overline{IA}$) is determined in the following way: In equilibrium $\overline{DI}=\overline{IA}$. Since $\overline{DI}=DF*\overline{EO}$ and $\overline{EO}=\overline{OD}$, it follows that $\overline{IA}=DF*\overline{OD}$, giving the initial value equation for IA

N IA=DF*OD

The values for the parameters are chosen

C F=0.25/DF=3/S=2/AD=8

The exogenous rate variable is OD. We assume that OD, in equilibrium, has a constant input of 500 units. The test input starting in period 5 should be a step function with a height of 100 units.

A OD.K=500+STEP(100,5)

Hence we obtain the DYNAMO program of model II:

```
L IA.K=IA.J+DT*(IO.JK-SA.JK)
N IA=DF*OD
R IO.KL=DELAY3(OM.JK,AD)
A DI.K=DF*EO.K
A EO.K=SMOOTH(OD.K,S)
R OM.KL=EO.K+F*(DI.K-IA.K)
R SA.KL=MIN(IA.K,OD.K)
A OD.K=500+STEP(100,5)
NOTE
C F=0.25/DF=3/S=2/AD=8
SAVE IA                               ┐
SPEC DT=1,LENGTH=200,SAVPER=4         │
RUN R1                                │
C F=0.2/S=4                           │ DC
RUN R2                                │
C F=0.06/S=2                          │
SPEC SAVPER=0,PLTPER=4                │
CPLOT IA.R1=1,IA.R2=2,IA=3            │
RUN                                   ┘
```

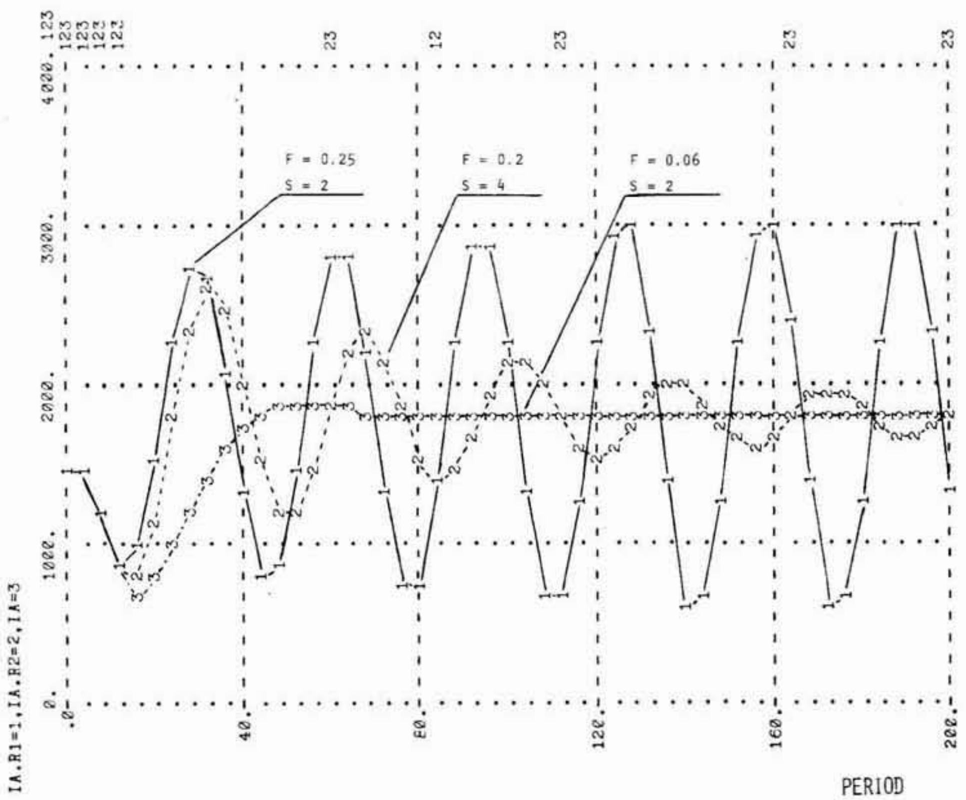


Fig. 10. Step response of IA in inventory and production planning model II with different parameters of F and S (DF=3)

The instructions marked with "DC" are direction cards which cause three simulation runs with computation and print plots of the IA step response (see [22, p. 44]).

4.2.2. Model Analysis and Modification

Figure 10 shows the simulation of the inventory level (IA) with different parameters of S and F and DF=3.

Clearly, the system has strong internally generated oscillations which can be reduced only with very small values of the parameter F. This raises the question of whether or not the decision rule which determines OM is suitable for achieving damped behavior. If not, it may be necessary to change the decision rule.

A closer examination of this model reveals that the decision rule is, in fact, unsatisfactory. Apparently, the policy for reducing the deviation between target and actual inventory does not take into account orders which are being processed but which have not yet been delivered.

A decision rule accounting for the backlog of orders is

R OM.KL=EO.K+F*(DI.K−IA.K+NOAS.K−AOAS.K)

NOAS – Normal orders in backlog (units)
AOAS – Actual orders in backlog (units).

NOAS is defined as:

A NOAS.K=EO.K*AD

and can be interpreted as the desired backlog of orders. The actual backlog of orders is

L AOAS.K=AOAS.J+DT*(OM.JK−IO.JK)
N AOAS=OD*AD

Figure 11 shows the step response of the modified model. The difference is impressive: there are no internally generated oscillations regardless of which of the three parameter combinations is used. In fact, the ordering policy with parameters F=0.25 and S=2, which previously (before the decision rule was changed) caused the strongest oscillation, is now the best of the three alternatives. This parameter combination produces the smallest downward dip before the system recovers equilibrium.

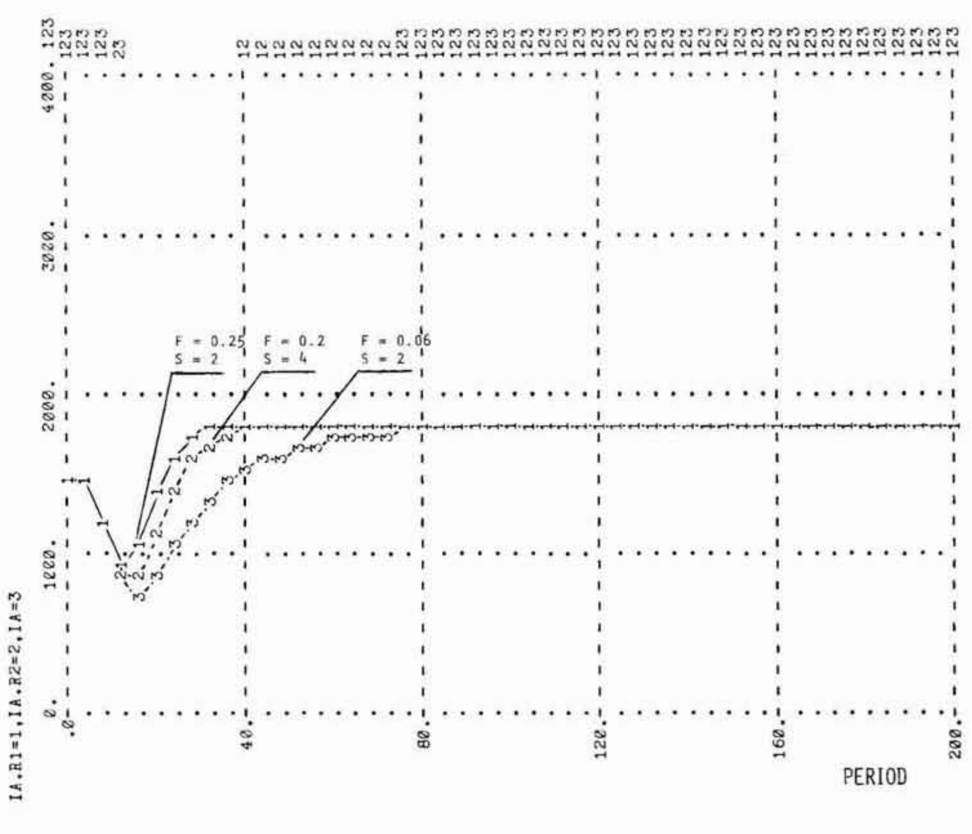


Fig. 11. Step response of IA in inventory and production planning model II after changing the decision rule for ordering

5. Analytical Investigation of Test Input Response Analysis in Linear System Dynamic Models

5.1. Analytical Methods for Investigating Linear System Dynamics Models

Most System Dynamics models are non-linear; this non-linearity is, however, of a low degree and in many cases non-linear System Dynamics models can be satisfactorily approximated by linear models. (The Sprague model which will be discussed in the next section was, for instance, linearized by Fey [9].) The analytic investigation of the structure of linear models yields constructive insights for evaluating the test input response analysis made in the System Dynamics approach.

In a linear, singularly open System Dynamics model it is possible to determine the so-called *final equation* for each variable (V). The general form of this equation (in conventional difference equation notation) is

$$V(t) = -a_1 V(t-1) - \ldots -a_n V(t-n) + b_0 T(t) + +b_1 T(t-1) + \ldots + b_m T(t-m) \quad (34)$$

T expresses the exogenous variable of the model. In the case of a test input response analysis with a step input of height $\overline{T}$, the solution of (34) is

$$V(t) = TR(t) + G \cdot \overline{T} \quad t=0,1,2,\ldots \quad (35)$$

G is the *gain factor* of V and TR is the *transient response term.* G is defined by

$$G = \overline{V}/\overline{T}. \quad (36)$$

Assuming the linear model (34) is in equilibrium, one has

$$\overline{V}(1+a_1+\ldots+a_n) = \overline{T}(b_0+b_1+\ldots+b_m) \quad (37)$$

allowing G to be expressed by the parameters of the model:

$$G = \frac{b_0+b_1+\ldots+b_m}{1+a_1+a_2+\ldots+a_n} \quad (38)$$

TR is the solution of the reduced equation of (34). It can be expressed:

$$TR(t) = C_1\lambda_1^t + C_2\lambda_2^t + \ldots + C_n\lambda_n^t, \quad (39)$$

where $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the (unequal) roots of the characteristic equation of (34), i.e.

$$\lambda^n + a_1\lambda^{n-1} + \ldots + a_{n-1}\lambda^1 + a_n\lambda^0 = 0. \quad (40)$$

The coefficients $C_1, C_2, \ldots, C_n$ are determined by (34) and (35) and depend on $a_1, \ldots, a_n, b_0, \ldots, b_m$, and $\overline{T}$.

The gain factor (G) and the transient response term (TR(t)) determine the behavior of a linear system. Hence the study of the behavior of a linear System Dynamics model can be reduced to the study of different amounts of G and different time trajectories TR. Let us first look at the gain factor. It shows how much the variable V increases if the exogenous variable T is increased by a step input of one unit and the system comes to an equilibrium. The determination of the gain factors in System Dynamics models is relatively easy. Taking as an example the is-ought decision rule (24) used by Forrester, it can be seen that, at equilibrium, the is-ought component must be zero. It then follows, that at equilibrium, the desired level (DL) must equal the actual level (L). From (25) follows $\overline{DL}=DF*\overline{EXSR}$. Since EXSR is the exponential smoothing of the rate variable R, this means that at equilibrium $\overline{EXSR}=\overline{R}$. The rate variable R is identical to the exogenous variable T, so $\overline{R}=\overline{T}$. It follows from these relations that

$$\overline{L}=DF\cdot\overline{T} \tag{41}$$

A comparison with (36) reveals that the depletion factor (DF) is the gain factor of the level variable L.

The transient response term describes a time trajectory. As the coefficients $C_1, \ldots, C_n$ do not change with t, the behavior of the transient response term depends only on the roots $\lambda_1, \ldots, \lambda_n$.

If at least one root in absolute value is greater than unity, the transient response term will, with increasing t, approach infinity, i.e. the system is unstable. In the case of an inventory and production planning system, this situation is not acceptable. Hence for realistic systems one must have satisfied

$$|\lambda_i| < 1 \quad i=1,2,\ldots,n. \tag{42}$$

Under this general condition of stability two ways of behavior are of interest. If the roots are real and satisfy the condition

$$0 < \lambda_i < 1 \quad i=1,2,\ldots,n$$

the system shows a monotonic damped behavior, i.e. the time trajectory of the step response approaches the new equilibrium without oscillations or fluctuations. If the system has no monotonic damped behavior, at least one component of the transient response term will show an oscillation or fluctuation.

In case the characteristic equation possesses a pair of complex conjugate roots, two terms of the transient response term are to be replaced by the trigonometric expression [see 16, p. 55]

$$r^t A[\cos(\varphi t-\theta)]. \tag{44}$$

This expression leads to an oscillation of the transient response term. Parameters r and A determine the amplitude of the oscillation, while the oscillation period is determined by:

$$D = 360/\varphi \quad \text{(periods).} \tag{45}$$

If one of the real roots is negative, then the transient response term has an alternating component which induces a fluctuation in the time trajectory.

A study of the system's behavior must include the investigation of the conditions under which the special kind of behavior occurs. If an inventory and production planning system shows an oscillating behavior, it is (as mentioned) our objective to avoid such oscillations, i.e. to realize a monotonic damped behavior. In the case of a linear system, this improvement can be realized by changing the controlled parameters of the model to satisfy condition (43).

5.2. Analytical Investigation of the Test Input Response of Inventory and Production Planning Model I

The analytical methods discussed above will now be used for the investigation of model I (Sect. 4.1). First, model I is transcribed into normal difference equation notation

$$P(t+1) = W(t)*PL$$
$$SS(t) = SS(t-1) + [S(t-1)-SS(t-1)]/SC$$
$$ID(t) = DF*SS(t)$$
$$WHD(t+1) = [WD(t)-W(t)]/TL$$
$$WD(t) = WAS(t) + WIA(t)$$
$$WAS(t) = SS(t)/PL$$
$$WIA(t) = [ID(t)-IA(t)]/[AF*PL]$$
$$W(t) = W(t-1) + WHD(t)$$
$$IA(t) = IA(t-1) + P(t) - S(t-1).$$

The final equations for W and P are:

$$\begin{aligned} W(t) &= -a_1\cdot W(t-1)-a_2\cdot W(t-2)-a_3\cdot W(t-3) + \\ &\quad +b_2\cdot S(t-2)+b_3\cdot S(t-3) \\ P(t) &= -a_1\cdot P(t-1)-a_2\cdot P(t-2)-a_3\cdot P(t-3)+ \\ &\quad +b_2^*\cdot S(t-2)+b_3^*\cdot S(t-3) \end{aligned} \tag{46}$$

$$a_1 = -3 + \frac{TL + SC}{TL \cdot SC}$$

$$a_2 = \frac{-2TL-2SC+1}{TL \cdot SC} + 3 + \frac{1}{TL \cdot AF}$$

$$a_3 = \frac{-1+SC+TL}{TL \cdot SC} - 1 - \frac{SC-1}{TL \cdot SC \cdot AF}$$

$$b_2 = \frac{(AF+DF+SC) \cdot PL}{TL \cdot SC \cdot AF} \qquad b_2^* = \frac{b_2}{PL}$$

$$b_3 = \frac{(1-AF-DF-SC)\cdot PL}{TL\cdot SC\cdot AF} \qquad b_3^* = \frac{b_3}{PL}.$$

The transient response term is, according to (39)

$$TR(t) = C_1\lambda_1^t + C_2\lambda_2^t + C_3\lambda_3^t. \tag{47}$$

The roots of the characteristic equation

$$\lambda^3 + a_1\lambda^2 + a_2\lambda^1 + a_3 = 0 \tag{48}$$

are

$$\lambda_1 = 1 - \frac{1}{SC}$$

$$\lambda_{2,3} = 1 - \frac{1}{2TL} \pm \frac{1}{2TL}\cdot\sqrt{1-\frac{4TL}{AF}} \tag{49}$$

Substituting a_1 to b_3^* in (38), the gain factor of P is found to be 1. This result is plausible since, in an equilibrium state, the production rate (P) must equal the sales rate (S), i.e. $\overline{P}=\overline{S}$. It was mentioned earlier that the depletion factor in an is-ought decision rule equals the gain factor. Therefore the gain factor of the variable "actual inventory" (IA) is equal to the inventory depletion factor (DF). Expression (49) shows that the roots are not dependent upon the inventory depletion factor (DF), i.e. the gain factor of IA. Therefore the dynamic behavior of the system is not dependent upon the inventory depletion factor (DF). By simulating the model, this proposition could be confirmed by many simulation runs with varying DF. But only through such an analytical investigation can we get an absolute guarantee that DF has no influence in any case.

Looking at the remaining parameters, we recognize that the smoothing time constant (SC) has no influence, whether the system has an oscillating or monotonic damped behavior. Since (with DT=1) the value of SC should always be chosen greater than 2 [22, p. 44], λ_1 will always be positive and less than unity. An oscillation of the transient response term can therefore only occur if the roots λ_1 and λ_2 of (49) are conjugate complex or negative. As λ_1 and λ_2 are dependant upon AF and TL, the numerical value of these parameters determines whether the system is oscillatory or monotonic damped. This feature of the system's behavior cannot definitively be explored by a (finite) series of simulation runs. In the case of such small models an experienced analyzer will soon come (by inductive reasoning using the simulation results) to the true conclusion that only AF and TL are responsible for an oscillatory behavior. A further investigation of AF and TL shows that these parameters lead either to a monotonic damped or to an oscillatory behavior, which is caused by a pair of complex roots.

Figure 12 shows which combinations of AF and TL leads to the different modes of behavior. Let us assume the original system's parameters of TL and AF are lying in the region of an oscillatory behavior. In that case, an improvement of the system's behavior is obtained by choosing a combination of AF and TL which fall within the area of a monotonic damped system.

Let us now investigate the 6 simulation runs of inventory and production planning model I which are shown in Table 2. The numerical values of the parameters of

Table 2. Different parameter values AF, SC, TL of inventory and production planning model I and the corresponding parameter values of the transient response term (47) of a step response of W and P

Case No.	Controlled parameters of model I			Roots of the transient response term (47)			Coefficients of the transient response term (47)			Trignonometric expression (44) of the components $C_2\lambda_2 + C_3\lambda_3$ in the transient response term (47)			
	AF	SC	TL	λ_1	λ_2	λ_3	C_1	C_2	C_3	r	A	D	θ[°]
1	8	2	2	0.5	0.75	0.75	225	−137.5	−137.5	–	–	–	–
2	10	6	3	0.8311	0.8345 −0.0746i	0.8345 +0.0746i	550	–	–	0.838	605	70.58	188
3	4	4	8	0.75	0.9375 −0.1653i	0.9375 +0.1653i	31,3	–	–	0.952	83	36	168
4	2	8	2	0.875	0.75 −0.433i	0.75 +0.433i	23	–	–	0.866	81.5	12	206
5	2	4	2	0.75	0.75 −0.433i	0.75 +0.433i	50	–	–	0.866	104	12	196
6	2	2	4	0.5	0.875 −0.3307i	0.875 +0.3307i	37,5	–	–	0.935	90	17.4	165

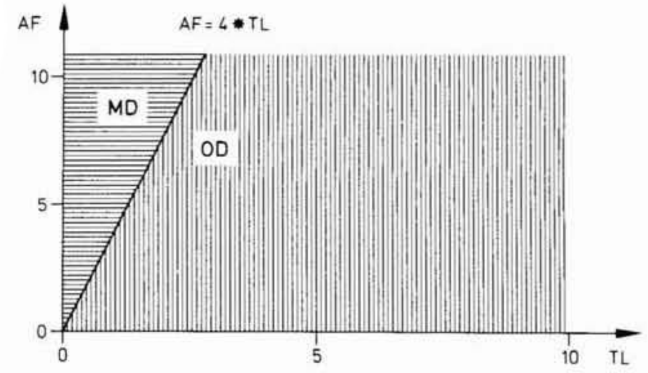


Fig. 12. Areas of monotonic damped (MD) and oscillatory damped (OD) behavior of inventory and production planning model II in relation to the parameters AF and TL

the transient response term are compared to the simulation runs of the inventory model. In cases 2 to 6, the roots λ_2 and λ_3 are conjugate complex. The parameters of the oscillating term (44) are listed in the columns on the left side of the table. We can see that the policy of simulation run 1, which was elected as the best system improvement, is the only case of a monotonic damped system behavior.

An analytic investigation of large, dynamic, linear models is very cumbersome and not advisable. The analysis in this section, however, shows which modes of behavior are possible even in larger models, and it demonstrates also the limitations of the test input response analysis to deduce general conclusions about the system's behavior.

The fact that the transient response term does not depend on the amount of the step input $\overline{T}$ is a typical feature of linear systems. Therefore in a non-linear system it is possible that one step input will produce an oscillatory behavior while a step input with a different amount will produce a monotonic damped behavior. Since System Dynamics models are predominantly non-linear, it is possible that a test input response analysis cannot give a complete picture of the system's behavior.

Before the problems of validation and implementation are discussed, the structure and analysis of an implemented inventory and production planning model will be described.

6. The Sprague Electric Company Model

6.1. Description of the Model

To gain further insights we shall examine a model designed by Forrester for the Sprague Electric Company. This model analyzes the production operations of a line of electronic components. (According to Sprague, "some hundred thousand dollars" were spent on the development of this model [29, p. 321].)

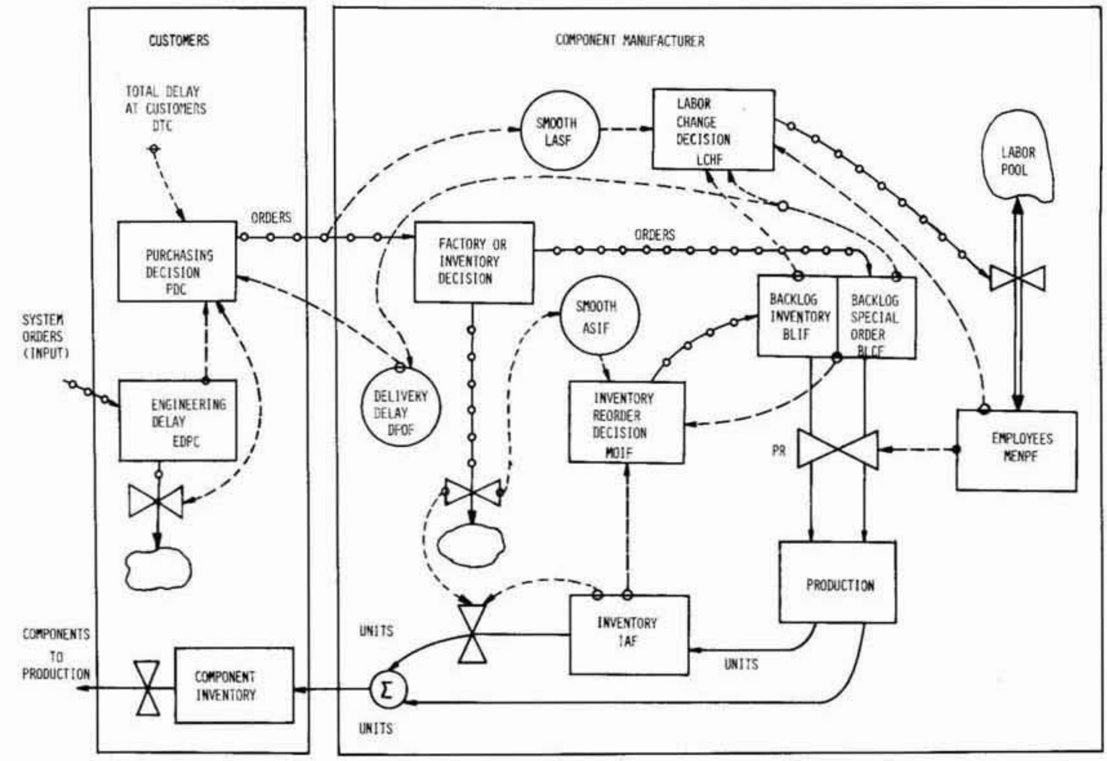


Fig. 13. Important structural relations of the Sprague Electric Company Model

The model includes both customers and company behavior. We shall first describe the main relationships, then examine each decision rule.[3]

Incoming customer orders are divided into two streams according to the "factory or inventory decision": one consisting of customer orders, which can be filled directly from inventory; the other, of orders which cannot be filled from inventory (i.e. must be specially produced). The latter forms the "backlog of special orders." The inventory, having been depleted by customer orders, is restocked by inventory management with the help of the "inventory reorder decision rule." These production orders compromise the "backlog of inventory orders." The rate of processing the two backlogs depends upon the production rate (PR) which in turn depends upon the number of men producing at factory (MENPF). MENPF is determined by the "employment change decision."

6.1.1. Purchasing Decision Rule

The "purchasing decision of the customer" (PDC) is determined by:

$$PDC.KL=ECPC.K/DEEDC.K \qquad (50)$$

PDC – Purchasing decision (units/week)
ECPC – Engineering design in process (units)
DEEDC – Delay effective in engineering department (weeks)

DEEDC is – in somewhat simplified terms – determined by the difference between the target value of the "total delay at customer" (DTC), and the "delivery delay at factory" (DFOF). DEEDC decreases as DFOF increases, which creates an increase in the "purchasing decision rate" (PDC). Since an increase in the purchasing decision rate – ceteris paribus – causes an increase in the delivery delay (DFOF), PDC shows self-induced growth.

6.1.2. Factory-or-Inventory Decision Rule

Items ordered which are in stock will be delivered immediately. The fraction of orders which can be filled from inventory is expressed by a non-linear table function. It is assumed that the fraction of orders which can be supplied from inventory increases with inventory size.

6.1.3. Inventory Reorder Decision Rule

Inventory management policy is directed by the inventory reorder decision. This is a linear is-ought decision rule:[4]

$$MOIF.KL = ASIF.K + \frac{1}{TIAF} * (IDF.K-IAF.K+OINF.K-OIAF.K) \qquad (51)$$

MOIF – Manufacturing orders for inventory (units/week)
ASIF – Average shipments from inventory (units/week)
TIAF – Time for inventory adjustment (weeks)
IDF – Inventory desired (units)
IAF – Inventory actual (units)
OINF – Orders for inventory desired (units)
OIAF – Orders for inventory actual (units)

OINF is determined by

$$OINF.K=ASIF.K*(DPF+BLIF.K/PIOF.K) \qquad (52)$$

DPF – Delay in production (weeks)
BLIF – Backlog for inventory (units)
PIOF – Production of inventory orders (units/week)

in which the term BLIF.K/PIOF.K is the (variable) delay in the backlog of inventory orders. As this term increases, OINF also increases and – ceteris paribus – therefore MOIF. Since MOIF is the inflow rate of the inventory backlog (BLIF), MOIF, by means of this positive feedback relation, induces its own growth.

6.1.4. Employment Change Decision Rule

Since in this model there is no over- or underemployment, the production rate (PR) is determined directly by the "number of men producing at factory" (MENPF). This level is controlled by the labor change decision (LCHF), a nested linear is-ought decision rule:

3 The model consists of 22 normal level equations, 24 rate equations, and 35 auxiliary equations and is described in [11]. One simulation run affords 0,86 s. (DT=0.25, LENGTH=100) CPU-time on an IBM 370/158 with DYNAMO II. Coyle has drawn attention to the surprisingly small number of studies examining the practical application of System Dynamics to inventory and production control [14, p. 445]

4 The necessary condition of nonnegativity of decision variables like MOIF, which must be guaranteed in the model by special instructions, is not expressed in this and the following equations

$$LCHF.K = \frac{1}{TLCF}\Big(\underbrace{LASF.K + \frac{BLTF.K - BLNF.K}{CPLF*TBLAF}}_{\text{Is-ought backlog}} \underbrace{- MEIPF.K}_{\text{Ought-working force}} \underbrace{- LTF.K + MENPF.K}_{\text{Is-working force}}\Big) \quad (53)$$

LCHF – Labor change rate for hiring (men/week)
TLCF – Time for labor change (weeks)
LASF – Labor for Average sales (men)
BLTF – Backlog total (units)
BLNF – Backlog desired (units)
CPLF – Productivity constant of labor (units/man-week)
TBLAF – Time for backlog adjustment (weeks)
MEIPF – Men for excess inventory production (men)
LTF – Labor in training (men)
MENPF – Men producing (men)

The rate of processing the backlogs for customer and inventory orders is given in the decision rule

$$PR.KL = \underbrace{\frac{BLCF.K}{DMBLF}}_{\substack{\text{production rate}\\ \text{for inventory}}} + \underbrace{\frac{BLCF.K}{DMBLF}}_{\substack{\text{production rate}\\ \text{for customer}}} + PEIF.K \quad (54)$$

PR – Production rate (units/week)
BLCF – Backlog for customer (units)
BLIF – Backlog for inventory (units)
DMBLF – Delay in backlog (weeks)
PEIF – Production excess for inventory (units/week)

If the production capacity MENPF.K/CPLF cannot satisfy the production rates specified by (54) for inventories and special customer orders, the rates are reduced correspondingly.

These are the essential relationships of the model. Forrester's model also includes a subsystem for raw material planning (controlled by an is-ought decision rule) which, however, has no decisive influence upon the system's behavior. The same is true of the subsystems for cash flow, profits and dividends which Forrester added after completion of the original model.

6.2. Analysis and Modification of the Sprague Model

Figure 14 depicts the step response of several variables of the model. The system shows damped oscillation behavior with employment (MENPC) having greater variation than incoming factory orders (RRFPC). The long-term cyclical pattern of the system is conditioned endogenously. An intensive study of the system reveals that the cause of both internal oscillation and its amplification can be narrowed down to a few decision rules. There is a cyclical shift in the amount of orders and the size of work force. If the delivery delay increases, the customer orders products immediately ("ordering ahead"), which he otherwise would have ordered later. The growing backlog of customer orders prompts the hiring of more employees. The resultant shortening of the delivery delay causes the customer to order less. The manufacturer then dismisses employees in order to decrease production capacity. This lenghtens the delivery delay, the customer orders ahead, and the same mechanism begins all over again. A self-perpetuating cycle is present, similar to agriculture's "pig cycle."

The recurrent nature of the fluctuation is one aspect of the problem; the other is that the degree of fluctuation in employment, inventories, and order backlogs is much greater than the fluctuation in incoming orders. One reason for this is the inventory ordering rule (51): an increase in incoming orders immediately occasions an increase in restocking orders to adjust inventory to a target level (which has been raised because of an increase in the incoming orders). This mechanism of self-induced growth is a positive feedback, which for OINF (added orders for inventory desired) is the result of a precipitate ordering-ahead policy. Instead of waiting until the peak of customer orders is over, inventory management amplifies this backlog peak by immediately reordering for inventory adjustment.

In (54), production rates for special customer orders and inventory orders, respectively, are determined proportionally in relation to their backlogs, (only) when the entire production capacity is engaged in filling orders. The delivery delay, which leads to cyclical shifting, is determined by the backlog of customer orders.

If the filling of special customer orders were given priority over filling inventory, the delivery delay would be reduced, thereby preventing the ordering ahead policy of the customer. Therefore the inventory order policy (51), in conjunction with the priority of order filling policy (54), are the primary causes of cyclical shifting.

The system was modified as follows to avoid amplification and shifting effects. The policy of immediate inventory adjustment as in (51) was eliminated. The priority decision for production (54) was changed so that, at all times, a fourth of the special orders backlog is being manufactured; the remainder of production capacity is used for inventory adjustment

$$PR.KL = \underbrace{CPLF*MENPF.K}_{PIF.K} - \underbrace{BLCF.K/4}_{PCOF.K} \quad (55)$$

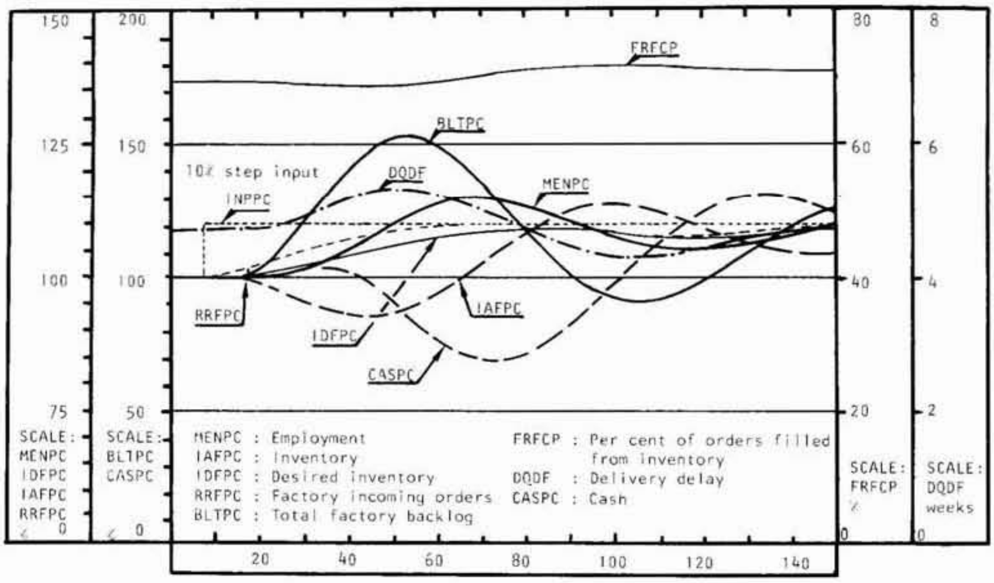


Fig. 14. Step response of some important variables in the initial Sprague Model

PR – Production rate starts for inventory (units/week)
CPLF – Constant, productivity of labor (units/man-week)
MENPF – Men producing (men)
BLCF – Backlog for customer (units)
PCOF – Production rate to customer order (units/week)
PIF – Production rate start for inventory (units/week)

The rate of inventory production is controlled by the number of men producing at factory (MENPF). MENPF is determined by the hiring and dismissal rate (LCHF). The redesigned decision rule is:

$$LCHF.K = \frac{1}{TLCF}\left[LASF.K + \frac{1}{TBLAF} * \underbrace{\left[\frac{1}{CPLF}\left(\overbrace{IDF.K - IAF.K}^{\text{Is-ought inventory}} + \overbrace{OINF.K - OPIF.K}^{\text{Is-ought orders in production}} + \overbrace{BLCF.K - BLNF.K}^{\text{Is-ought order backlog}}\right)\right]}_{\text{Ought-work-force}} - \underbrace{LTF.K - MENPF.K}_{\text{Is-work force}}\right] \qquad (56)$$

LCHF – Labor change rate for hiring (men/week)
TLCF – Time for labor change (weeks)
LASF – Labor for average sales (men)
TBLAF – Time for backlog adjustment (weeks)
CPLF – Constant productivity of labor (units/man-week)
IDF – Inventory desired (units)
IAF – Inventory actual (units)
OINF – Orders for inventory desired (units)
OPIF – Orders in process for inventory (units)
BLNF – Backlog desired (units)
BLCF – Backlog for customer (units)
LTF – Labor in training (men)
MENPF – Men producing (men)

In contrast to the original decision rule (52), OINF is now determined by ASIF times DPF only. The new decision rule (56) for LCHF results in the adjustment of the actual inventory value to the target value (IDF). A simulation using this new decision rule shows that this adjustment occurs at the peak of the incoming order rate, thus amplifying the fluctuation of total incoming production orders.

By choosing a larger constant for exponential smoothing of the incoming orders (which determines target inventory IDF) and a larger adjustment factor (TBLAF) in (56), it was possible to shift the peak of the inventory adjustment more in the direction of lull in the incoming customer orders. The effect was a contracyclical load of the work force for inventory filling and special customer order production which leads to stronger damping of work force fluctuation. The step response of the redesigned model shows a fundamentally changed behavior (Fig. 15):

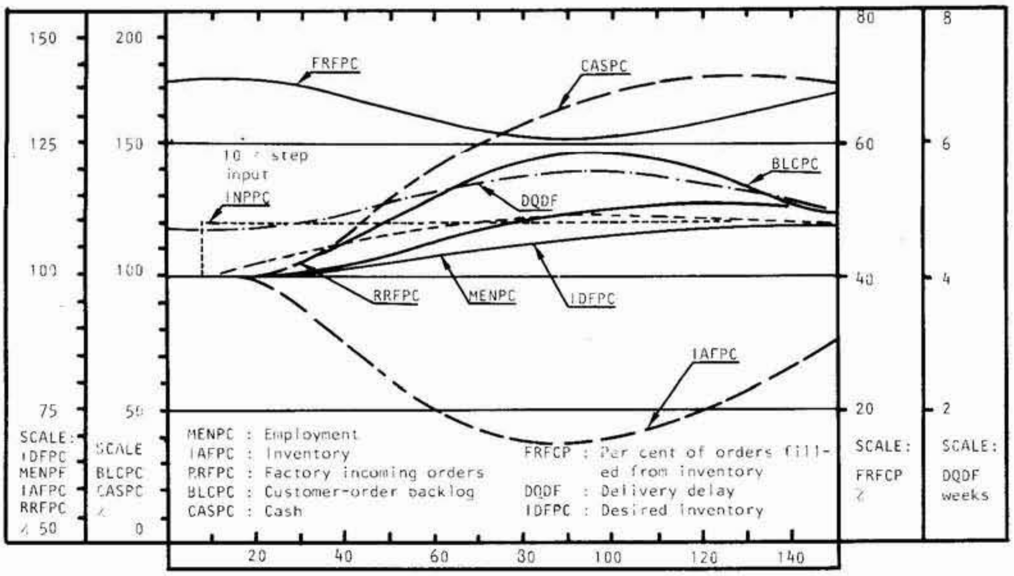


Fig. 15. Step response of some relevant variables in the redesigned Sprague Model

7. Validation of System Dynamics Models

Forrester maintains that a validation of his models according to conventional statistical criteria is not necessary. Although singularly open models (like the Sprague model) can, in principle, be judged for validity by ex-post forecasting, Forrester rejects this procedure, arguing that these models are not intended for forecasting but for studying and improving system behavior. The Sprague model is, he says, empirically valid because "it creates a pattern of the same qualitative nature as the real system even in the presence of a constant final demand for the product," i.e. also in case of a step input [14, p. 58]. The criteria, "a pattern of the same qualitative nature," cannot be objectively defined, insists Forrester. Whether a model is acceptable or not is up to the subjective judgement of the model user. Forrester has been attacked for these totally subjective criteria of validity [14, p. 81], [1].

According to Forrester, a System Dynamics model is merely an explication of the decision maker's mental model. One may ask, however, if mental models are of such a precise nature that they can be represented by a numerical mathematical model. For instance, it is highly improbable that a decision maker could, without using statistical methods, succeed in identifying a delay as an exponential third order delay or know its average delay. This is especially true because the form of the pulse response function (19) cannot in any way be observed directly.

Forrester's complete rejection of binding objective criteria for determining model validity is not shared by other designers of System Dynamics models. Schlager, for example, developed a complex inventory and production model for a manufacturer of liquid flow measuring instruments. He tested the validity of the model by means of an ex-post forecast [25, p. 146]. Statistical methods are also being used more frequently for hypothesis testing and validation of System Dynamics models in areas other than inventory and production [see 33]. (Peterson has developed a statistical software package for parameter estimation (GPSIE) on full-information, maximum likelihood via optimal filtering [21]. A FORTRAN parameter fitting procedure for third order delays is described in [34].)

A further criticism concerns the "infinitesimal approach" used by Forrester. In the models examined here, a value of DT=1 has been chosen to enable a plausible interpretation of the behavioral equations involved. In Forrester's view, however, DT should ideally approach zero. This means that a System Dynamics model should always be a differential equation model which would have absurd consequences in the case of inventory and production models. Let us consider, for example, the decision rule (56) for the hiring and dismissal of production workers. If the value of DT is chosen to be DT=1, the length of time between J and K is one week. As Carlson reports, this decision rule is, in fact, put into effect once a week by the Sprague company [3, p. 16]. However, Forrester had specified DT=1/20 (per week) for the model. This means that the model calls for a hiring-firing decision every $7 \cdot 24/20 = 8.4$ h. In an ideal case, moreover, DT would be infinitely small; only the need for an increase in computer time kept Forrester from specifying an even shorter DT than he did. In other words an "ideal" model of the Sprague company would call for decisions about employment, production rate, etc. to be made at infinitely small time intervals. What may be a valid practice in demographic models presents, in production and inventory models, problems of interpretation, needless waste of computer time, and distortion of the hypotheses which the decision rules express. (Since using a value of $DT \to 0$ has proven implausible in the case of inventory and production models, the following examples of System Dynamics models will continue to have DT=1.)

8. Implementation of System Dynamics Models

Implementation concerns the practical purposes for which a System Dynamics model should be used and the way in which these purposes can be achieved. According to Weil, we can distinguish between four stages of implementation [32].

"1. Change of the view of those involved in the project. 2. Broader impact on opinion. 3. Results used to support decisions. 4. Model adapted as a working tool."

As opposed to Weil, Forrester makes no explicit statement on the goal of implementation. He says the goal of building a model is "to understand the reality better" [10, p. 3–5] and to "get a better intuitive feeling for the time-varying behavior of industrial and economic systems" [13, p. 28]. According to Forrester, "some of the most useful insights to come from industrial dynamics show which policies in a system have enough leverage so that by changing them one can hope to alter system's behavior" [15, p. 141]. Whether this system improvement should be realized throughout by implementing the new decision rule in the system remains unanswered. Nevertheless, Forrester says about the Sprague model:

"All of the changes made in the system model in this chapter are one that can be readily made in the actual system. Their implementation requires the formalizing of critical policies of system control to ensure that they are consistently and routinely executed" [11, p. 308]. Since the new decision rules found in the Sprague model have been implemented, the fourth stage of Weil's implementation hierarchy has been achieved in this case.

9. An Appreciation of System Dynamics Models in Inventory and Production Planning

First, we shall ask whether system Dynamics is not too costly and time consuming for improving inventory and production systems. Then we shall consider whether the search for damping policies in production and inventory systems is, in general, a desirable objective.

9.1. Overkill Effects of System Dynamics Models

The first criticism concerns the practicality of System Dynamics models to damp internal oscillation in inventory and production systems. One can contend that the Sprague model represents a case of overkill, i.e., is such a gigantic model necessary to convince inventory management that their ordering policy needs improvement? Certainly this would have been evident to anyone with experience in inventory and production planning. Forrester argues that model building is necessary since social systems often behave counterintuitively. By this he means the logical consequences of an assumption may conflict with the intuitive conclusions one might draw.

Thus a mathematical model representing the mental model is necessary to deduce the logically valid but counterintuitive consequences of the mental model. It is questionable whether this counterintuitive argument really applies in the field of damping inventory and production systems. Rosenkranz reports, for instance, that detection of any internally generated oscillation in the Ciba Geigy firm would have been possible "independently from the model by a careful analysis of order time series" [24, p. 337].

9.2. Goals for the Improvement of System Dynamics Models

9.2.1. Damping of Internal Oscillations as the Primary Goal

If it is the supreme objective of implementation that the model user gets a better intuitive feeling of the system (Stage 1 of Weil), then nothing can be said about the success in improving the real system. It is hardly possible to make a general forecast of how a better intuitive feeling of a manager leads to a system improvement. Therefore we assume for the following discussion that the objective is to realize stage 4 of Weil's list. This is using a System Dynamics model (as the Sprague model) to find better decision rules in order to implement them in the system. Under this assumption, the objective for an improvement of an inventory and production planning model is the damping or elimination of internally generated oscillations.

This would mean that the use of System Dynamics models in inventory and production planning would be restricted to those systems in which such behavior can be observed.

It is difficult, however, to decide which existing system falls into this category. A single case which illustrates this problem is documented by the Ciba Geigy report. This firm developed a System Dynamics model of its production and inventory system. The model, however, was not implemented, for "the analysis showed that the inventory cycles were mostly induced by fluctuations of external demand and were not internally generated" [24, p. 337].

Assuming Forrester's method is used on a system which does have internally generated oscillation, the goal of reducing this oscillation behavior cannot, even in this case, serve as the primary goal of inventory and production planning. Rather, the actual goal of inventory and production planning must be the introduction of those decision rules which reduce or minimize inventory and production costs.

What follows then is an evaluation of the effectiveness of Forrester's method in achieving a minimization of inventory and production costs. To develop a System Dynamics model of an inventory and production system, it is, in principle, not necessary to know the function of the inventory and production costs. Neither is it necessary to make a forecast of the exogenous variable or its probability distribution. However, an evaluation of Forrester's method differs whether or not such information about cost functions and exogenous variables is available.

9.2.1.1. Damping Internal Oscillation when the Cost Function is Unknown and no Forecast of the Exogenous Variable is Known. Damping the internal oscillation of a system may lead in many cases to a reduction of costs, because the cost of changing the production rate and work force level are thereby reduced. So if the function of the inventory and production costs is unknown, and a reasonable forecast of the exogenous sales variable does not exist, then the use of Forrester's method may be recommended for improving oscillating systems. Such, apparently, was the situation in the Sprague company case. As Carlson reported about the project, "no attempt has been made to measure exactly the effect on profits of the new policies. This would be a very difficult and costly undertaking requiring extensive changes in company-wide accounting systems ..." [3, p. 142].

The usefulness of Forrester's damping policy for reducing costs will be demonstrated in the following example. Holt, Modigliani, Muth, and Simon developed a planning model of a paint factory (HMMS-model). The authors determined the cost function of the inventory-production and work force sector [17]:

$C(t) =$

$340W(t)$	Regular payroll costs	
$+ 64.3[W(t) - W(t-1)]^2$	Hiring and layoff costs	
$+ 0.2[P(t) - 5.67W(t)]^2$ $+ 51.2P(t) - 281W(t)$	Overtime-undertime costs	
$+ 0.0825[IA(t) - 320]^2$	Inventory and shortage cost	(57)

with

C – Total cost ($/month)
W – Work force (men)
P – Production rate (units/month)
IA – Inventory (units)

The authors attempted to find for the work force (W) and production rate (P), decision rules which would minimize the expected value of C for a given planning horizon. Actual inventory (IA), which in the case of negative values represents the backlog of unfilled orders, is defined:

$$IA(t) = IA(t-1) + DT[P(t)-S(t)] \tag{58}$$

S – Sales (units/month)
DT – Time increment (month) [DT=1]

The structure of the model proved to be a special case in the theory of optimal multi-stage decisions for which an optimal decision rule could be calculated in the form of a linear function. For the goal function:

$$\sum_{t=0}^{\infty} E\{C(t)\} \rightarrow \min$$

the authors calculated the optimal linear decision rules:

$$\begin{aligned} W(t) &= 0.0455E\{S(t)\} + 0.742W(t-1) - \\ &\quad -0.00996IA(t-1) + 2.003536 \\ P(t) &= 0.8224E\{S(t)\} + 1.005W(t-1) - \\ &\quad -0.464IA(t-1) + 153.12 \end{aligned} \tag{59}$$

In the goal function, $E\{C(t)\}$ is the expected value of the total costs C(t), and $E\{S(t)\}$ is the expected value of the sales S.

Assume that a System Dynamics model is to be made for the production and inventory sector of this paint factory. We assume further that neither the cost function (57) nor the stochastic description of S are known to the model designer.

The decision rules to determine the work force (W) and the production rate (P), are assumed to be the same as those used in the Sprague model. These same is-ought decision rules were used in inventory and production planning model I. Work productivity (PL) was assumed to be constant in the Sprague model [11, p. 229], and it will be assumed to be constant in the paint factory as well. The overtime-undertime costs in (57) are thus zero, since every worker is working at full capacity. Given these conditions, a work productivity factor of PL=5.2 (units/man) can be calculated from the term of overtime-

Table 3. Expected value of the average unit cost, as affected by use of typical System Dynamics decision rules for inventory-production and work force planning in a paint factory

Case Nr.	Parameters			Expected value of average cost [$ per unit]			
	AF	SC	TL	DF= 1.0	DF= 0.8	DF= 0.64	DF= 0.25
1	8	2	2	76.65	71.92	70.53	76.03
2	10	6	3	78.26	73.03	71.23	75.80
3	4	4	8	78.29	73.47	72.02	77.40
4	2	8	2	82.60	77.39	75.63	80.38
5	2	4	2	87.42	81.86	79.84	83.94
6	2	2	4	82.48	77.31	75.58	80.37

undertime costs in (57). Since this is the same work productivity as that chosen in model I, the six cases in Fig. 9 show the different step response characteristics of the paint factory model, i.e., the System Dynamics version of the paint factory is identical with model I (except for inclusion of the cost function (57)). Table 3 shows these six different policies with various inventory depletion factors (DF) compared with the expected unit costs as determined by cost function (57) and a special stochastic specification of S.[5]

Comparing Fig. 8 with Table 3 we see that an increase in the damping of the system will decrease the expected values of the average unit cost. Thus the objective of damping the internal oscillations largely conforms to the objective of reducing costs.

9.2.1.2. Damping Internal Oscillation when the Cost Function is Unknown and a Forecast of the Exogenous Variable is Known. If the probability distribution (or some of its parameters) of the exogenous sales variable is known, then it should be asked if other goal variables will lead to better conformity to the goal of reducing costs. For instance, if the stochastic characteristics of the exogenous variable S are known, it is possible to determine the time path of the expected value and the standard deviation of the production rate (P) and the inventory (IA). The time path of the standard deviation can provide a better measure of system oscillation (and thus the existence of change costs) than the selection of "improved" models by test response simulation.

Other criteria are conceivable too. In a servo theory analysis of an inventory control system, Vassian developed two decision rules, to achieve the best possible forecast of the minimization of inventory variations resulting from forecast errors [31]. Similarly Elmaghraby determined the parameter of a decision rule resulting from the variance of the stochastic variable describing the deviation between desired and actual inventory.

9.2.1.3. Damping Internal Oscillation when the Cost Function and the Forecast of the Exogenous Variable are Known. If an information situation is at hand in which both the cost function and the probabilistic structure of the exogenous variable S are known, then a reasonable goal is to minimize the expected value of inventory and production unit costs. Test input response analysis, which requires less information, can play here only a subsidiary or supporting role.

Minimization of inventory and production costs is achieved by models which deal with the so-called production smoothing and work force balancing problem.[6] The analytical solution of such models, i.e., the determination of optimal decision rules, is possible only to a limited extent. Nevertheless such models can be simulated using different decision rules, and in this way an optimum can be approximated.

If S is merely an estimation of the expected value time path of S, use of these estimated values creates a deterministic model. Also in this case it may be possible to reduce the total average costs through simulation involving different decision rules, variation of parameters, and redesign of model sections.

Since for large, multi-stage, deterministic, and stochastic models there is no practicable optimization algorithm available, one should simulate these models, but not using test response simulation. Rather, the primary goal should be the minimization of the cost function.

9.2.2. Reduction of Production and Inventory Costs as the Primary Goal

Forrester's only goal in the analysis of production and inventory systems is apparently the damping of internal oscillation. Other designers of System Dynamics models of inventory and production planning, however, appear to place more value on cost reduction.

Schlager, in his previously mentioned model, included a considerably more developed cost sector. As he remarks, "all known costs related to production-inventory-employment were included to evaluate the cost benefits of greater stability to management and to existing system" [25, p. 147].

Thus it is clear that in this model the costs and not the system damping represent the primary target. Oertli-

5 The expected average unit costs are calculated for a planning time of 50 periods. These were estimated on the basis of 50 simulation runs, and a distribution of S: $S \in N$ ($\bar{x}$ = 500, σ = 75)

6 See [2], [6], [27].

Cajacob's System Dynamics model, which describes a four-stage inventory and distribution system, serves as an example of such a procedure [20, p. 320]. His model includes a strongly differentiated cost sector. In the model simulation, Oertli-Cajacob assumes different organisational changes in the degree of centralization. As goal functions he uses the average yearly costs and the average order-filling ability. It seems as if the development and redesign of System Dynamics models, with the primary goal being the reduction of the unit costs, represents a variant of the production smoothing and work force balancing problem. This shall be investigated now.

9.2.2.1. Cost Reduction in System Dynamics Models through Use of Test Input Response Analysis. If in a System Dynamics model the unit costs should be reduced by a better decision rule, test input response analysis is not the appropriate method. The simulation of different decision rules should not start from a model equilibrium, and the time path of the exogenous variable should not be a test input. Rather, in a deterministic model, S should represent a forecast of S for the planning period, and in a stochastic model S should be described by a sequence of pseudo random numbers belonging to the probability distribution of S. To judge the unit cost by carrying out a test input response analysis is not a reasonable procedure.

When cost reduction is the primary goal of the System Dynamics Model, one should not analyse and redesign the model using a test input response analysis.

9.2.2.2. Cost Reduction in System Dynamics Models using Is-Ought Decision Rules. As we have seen, Forrester uses is-ought decision rules to control certain level variables such as inventory or work force. The class of is-ought decision rules is, however, only one of the many types of decision rules which are in principle possible. Therefore the danger exists that, by restricting decision rules to the is-ought type, the cost optimal decision rules will be excluded from the model.

The above paint factory model will serve to illustrate this. Calculation of the final equation of the optimal decision rules according to the HMMS-model gives:

$$W(t) = 1.278W(t-1) - 0.407W(t-2) + 0.013E\{S(t)\} + 0.0099S(t-1) - 0.595 \quad (60)$$

$$P(t) = 1.278P(t-1) - 0.407P(t-2) - 0.464S(t-1) - 0.334S(t-2),$$

where $E\{S(t)\}$ is the expected value of $S(t)$. The final equations of the System Dynamics model of this system are given in (46).

A comparison of the decision rules reveals that there is no parameter combination of IR, TL, SC, and AF by which (46) becomes the optimal final Eq. (60). This means that, in this example, use of only is-ought decision rules has excluded the optimal case. Using the optimal decision rule (59) the expected value of the unit costs is 63.13[$/unit], which is considerably lower than the unit costs in Table 3, which are the result of using an is-ought decision rule. The decision rules (59) cannot be interpreted as is-ought decision rules. They express only the way in which P and W are to be chosen depending upon certain state variables, so that the unit cost is minimized.

Moreover, Schneeweiß has shown that linear decision rules (and a fortiori linear is-ought decision rules) yield, when used with non quadratic cost functions, in most cases only suboptimal strategies [26]. In many cases non-linear policies such as (s,S), (s,q), or (z,q)-policies yield better results. (See for these policies [19].) None of these inventory policies can be interpreted as is-ought decision rules, which have the goal of maintaining an explicitly given target inventory. Strategies which optimize stochastic dynamic decision models can be shown by dynamic programming to have the form:

$$x_{op}(t) = F_t[S_1(t-1), S_2(t-1), \ldots, S_n(t-1)], \quad (61)$$

where x_{op} is the optimal policy variable and S are the state (or level) variables of the system. This optimal decision rule (or strategy) has no restriction in respect to an explicitly stated is-ought structure. Following Forrester's proposal, the alternative strategies are restricted in the case of the often used decision rule (26) to the form:

$$\text{RAT.KL=SO.K}+\frac{1}{\text{APT}} * (\text{R1}*\text{SO.K}-\text{S1.K}+\ldots \ldots+\text{RN}*\text{SO.K}-\text{SN.K}), \quad (62)$$

where SO to SN are (some or all) state variables of the system. Forrester's is-ought decision rule seems to be a "paradigm" which originated from classical servomechanism theory. Modern control theory has, however, revealed the evident concept of is-ought decision rules to be too restrictive. One must not forget to consider this factor when attempting to find a reduced value of the cost function by the simulation of is-ought decision rules in a System Dynamics model.

10. Conclusion

The attempts in the last section to raise some points of criticism should not be seen as an attack upon the value of System Dynamics. Forrester's level-rate interpretation and the feedback view of systems, combined with the diagram technique of model representation make System Dynamics a heuristically fertile method of problem structuring and model building.

The System Dynamics diagram often provides a valuable basis for sharpening the precision of model relations which are necessary to construct the actual DYNAMO model. In the DYNAMO language the user has a very flexible and elegant tool for simulating models. The System Dynamics approach enables employees to learn very quickly a technique of describing and modelling the organisational relations in their firm. Even an attempt to develop a System Dynamics model often has positive consequences. Rosenkranz, for example, commended the usefulness and the "descriptive power" of a System Dynamics model of production planning, although this model has not been implemented [24, p. 337]. Considering the fact that in many firms the organisation must be described as far from ideal, the development of a System Dynamics model is often the vehicle for a thoroughgoing analysis of the system, frequently revealing "obvious" structural shortcomings which had previously gone unrecognized. A good example of this is the ordering ahead policy of the Sprague company which could probably have been recognized by a conventional organisational analysis but was factually realized by a System Dynamics study. System Dynamics has been approved as a valuable contribution in the field of dynamic model building and analysis.

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